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Benchmark calculations for higher-order parabolic equations

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Benchmark solutions generated with parabolic equation (PE) models are presented for range-dependent underwater acoustic propagation problems involving both penetrable and perfectly reflecting ocean bottoms. The solution of the wide-angle PE of Claerbout [J. F. Claerbout, *Fundamentals of Geophysical Data Processing* (McGraw-Hill, New York, 1976), pp. 206–207] agrees with the outgoing coupled-mode solution for the problems involving penetrable bottoms. The solution of the higher-order PE of Bamberger *et al.* [Bamberger *et al.*, "Higher Order Paraxial Wave Equation Approximations in Heterogeneous Media," SIAM J. Appl. Math. **48**, 129–154 (1988)], which is a generalization of Claerbout's PE, agrees with the outgoing coupled-mode solution for problems involving large variations in sound speed and propagation nearly orthogonal to the preferred direction. The computer code FEPE was used to generate the benchmark solutions and was found to run several times faster than the IFDPE computer code due to a tridiagonal system solver in FEPE that is optimized for range-dependent problems.

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INTRODUCTION

The accuracy of the parabolic equation[1] (PE) method in underwater acoustic modeling has been assessed with numerous range-independent benchmark problems.[2] The wide-angle PE[3] has performed well in most of these tests. Several range-dependent benchmark problems were recently posed and preliminary results were presented.[4] Some of these problems involve perfectly reflecting ocean bottoms and thus provide an extreme test of the ability of a propagation model to handle wide-angle propagation.

Since the standard wide-angle PE cannot handle propagation angles much larger than 40 deg, a higher-order PE model[5,6] based on a Padé series has been developed to handle these problems. The higher-order PE accurately handles propagation nearly orthogonal to the preferred direction and produces solutions essentially identical to the outgoing coupled-mode solution.[7] The computer code FEPE[8] is used to generate the benchmark solutions. An efficient tridiagonal system solver (not based on the standard Gaussian elimination scheme) in FEPE is discussed, and FEPE is found to run several times faster than the IFDPE[9] code for one of the benchmark problems. Solutions generated with the coupled normal-mode model COUPLE[10] are discussed.

I. THE HIGHER-ORDER PE MODEL

Solutions generated with PE models approximate the solution of the outgoing wave equation

$$\frac{\partial Q}{\partial r} = ik_0\sqrt{1+x}\,Q, \tag{1}$$

$$x = k_0^{-2}\left(k^2 - k_0^2 + \frac{\partial^2}{\partial z^2} - \frac{1}{\rho}\frac{\partial \rho}{\partial z}\frac{\partial}{\partial z}\right). \tag{2}$$

We refer to Eq. (1), which can be solved in terms of outgoing coupled modes, as PE_∞. The reference sound speed is c_0 where $k_0 = \omega/c_0$, r is the range from a point source, z is the depth below the ocean surface, k is the complex wavenumber, ρ is the density, and ω is the circular frequency. The PE field $Q(r,z)$ satisfies homogeneous boundary conditions at the top and bottom boundaries of the waveguide and the initial condition

$$Q(0,z) = \sqrt{2\pi i}\sum_j \frac{\phi_j(z_0)\phi_j(z)}{\sqrt{k_j}}, \tag{3}$$

where z_0 is the source depth. The normal modes ϕ_j and eigenvalues k_j satisfy

$$\frac{d^2\phi_j}{dz^2} - \frac{1}{\rho}\frac{d\rho}{dz}\frac{d\phi_j}{dz} + k^2\phi_j = k_j^2\phi_j. \tag{4}$$

In practice, either the Gaussian PE starter[1] or Greene's wide-angle PE starter[11] is often used to approximate $Q(0,z)$. In range-independent environments, the complex pressure P is related to Q by $Q \sim \sqrt{r}P$ for $k_0 r \gg 1$. In range-dependent environments, $\partial/\partial r$ and x do not commute, reflections can be generated, and a term involving $\partial\rho/\partial r$ appears in the wave equation. Thus $Q \sim \sqrt{r}P$ in range-dependent environments only if the range dependence is weak.

Bamberger *et al.* used a Padé series to approximate the square root in Eq. (1) and derive the following higher-order PE:

$$\frac{\partial U_n}{\partial r} = ik_0 \sum_{j=1}^{n} \frac{a_{j,n}x}{1+b_{j,n}x}U_n, \tag{5}$$

$$a_{j,n} = [2/(2n+1)]\sin^2[j\pi/(2n+1)], \tag{6}$$

$$b_{j,n} = \cos^2[j\pi/(2n+1)], \tag{7}$$

where $Q \cong Q_n = U_n \exp(ik_0 r)$. Equation (5), which we refer to as PE_n, has been applied to underwater acoustics and solved with the method of alternating directions.[6] This ap-

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proach involves n steps with the jth step requiring the solution of the equation

$$(1 + b_{j,n}x)\frac{\partial U_n}{\partial r} = ik_0 a_{j,n} x U_n. \tag{8}$$

Since Eq. (8) is of the same form as Claerbout's equation (or PE_1), for which simple and effective numerical solutions have been derived,[11-13] the alternating directions solution is easy to implement into an existing PE computer code.

The computer code FEPE solves PE_n using finite elements for depth discretization and Crank-Nicolson integration in range as described in Ref. 6. The tridiagonal system solver in FEPE has been designed to minimize computation time. The code uses an elimination scheme that involves sweeping downward to the row corresponding to the ocean bottom to eliminate entries below the main diagonal and sweeping upward to the ocean bottom to eliminate entries above the main diagonal followed by back substitution sweeping up and down from the ocean bottom. In contrast, Gaussian elimination involves sweeping downward to eliminate all entries below the main diagonal followed by back substitution sweeping upward.

For problems involving range-dependent ocean depth, the new scheme is more efficient than Gaussian elimination. In the decomposition into upper and lower triangular matrices of Gaussian elimination, it is necessary to repeat sweeping downward from the ocean bottom as the ocean depth varies. With the new scheme, it is necessary to repeat sweeping only for a few rows near the ocean bottom. Since multiplication is faster than division on computers, the tridiagonal system solver has also been improved by replacing divisors with factors. The code FEMODE[8] determines the eigenvalues using the finite-element matrices and constructs $Q(0,z)$.

II. BENCHMARK PROBLEMS AND RESULTS

Problem 1 consists of three parts each involving a wedge-shaped ocean in which the sound speed is 1500 m/s, the ocean depth decreases from 200 m to zero over the first 4 km from the source, and the surface is pressure release. A 25-Hz source is placed 100 m below the surface. For part A, a line source is used with a pressure release ocean bottom in plane geometry. For parts B and C, a point source is used in cylindrical geometry with sound speed 1700 m/s and density 1.5 g/cm^3 in the half-space sediment. The sediment is lossless for part B. The sediment attenuation is 0.5 dB/λ for part C.

Since energy is reflected back toward the source by a pressure release ocean bottom, PE_∞ cannot provide the full-wave solution for part A. However, we apply PE_n to this problem to show that it accurately handles the outgoing solution, which involves propagation angles up to nearly 90 deg near mode cutoff. PE_n is solved over a sequence of stair steps that approximate the wedge geometry. For this problem, it is necessary to remove reflected energy by mollifying[14] Q at the beginning of each stair step as follows:

$$Q(z) \rightarrow \int Q(z') \sum_{j=1}^{N} \phi_j(z')\phi_j(z)dz', \tag{9}$$

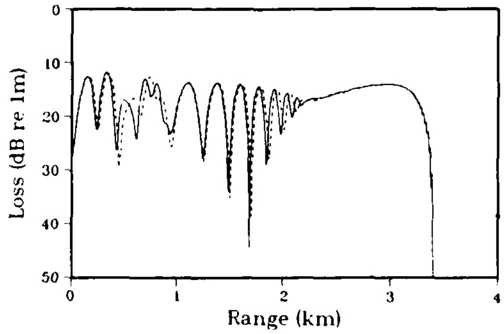

FIG. 1. The wedge with pressure release bottom in plane geometry. Transmission loss at depth 30 m. The solid curve is the PE_3 result. The dashed curve is the PE_1 result, which has large phase errors just before the mode cutoff ranges near 1 km and 2.2 km.

where the sum is over the N propagating modes at the range of the stair step.

Transmission loss at $z = 30$ m generated with PE_1 and PE_3 appears in Fig. 1. The PE_3 result agrees well with the PE_∞ result of Ref. 15. The PE_1 result exhibits the largest phase errors just before mode cutoff at $r = 1$ km and 2.2 km (three modes are excited by the source). Data for this, as well as the problems that follow, appear in Table I, in which CPU_n is the run time required by FEPE to solve PE_n on a Digital VAX-8650 computer, Δz and Δr are the depth and range increments, z_M is the maximum depth of the computational domain, and N_0 is the number of modes used to compute $Q(0,z)$.

For parts B and C, Greene's wide-angle PE starter is used, and the attenuation increases artificially in the lower portion of the sediment to prevent reflections from the artificial pressure release boundary at $z = z_M$. Transmission loss at $z = 30$ m and 150 m generated with PE_1 and PE_2 appears in Figs. 2 and 3 for parts B and C. The PE_1 results agree with the PE_1 results of Ref. 15 obtained using the IFDPE code, and the PE_2 results agree fairly well with the PE_∞ results of Ref. 15. This suggests that Greene's wide-angle PE starter is accurate for larger angles than PE_1. Using the input parameters used in Ref. 15, FEPE runs several times faster than IFDPE for this problem due to the efficient tridiagonal solver in FEPE.

The coupled-mode code COUPLE was also used to study this problem. Benchmark solutions generated with this model are not presented here, however, because this is done in Ref. 15. However, it is perhaps worth mentioning

TABLE I. Data for the benchmark calculations. Many of the input parameters are identical to those used in Ref. 15.

Case	N_0	c_0	Δr	Δz	z_M	(n,CPU_n)	(n,CPU_n)
1A	3	1500 m/s	5 m	1 m	···	(3,15 s)	(1,8 s)
1B	···	1500 m/s	5 m	1 m	4 km	(2,2 min)	(1,1 min)
1C	···	1500 m/s	5 m	$\frac{1}{2}$ m	2 km	(2,2 min)	(1,1 min)
2A	10	1700 m/s	1 m	$\frac{1}{2}$ m	$\frac{1}{2}$ km	(2,8 min)	(1,4 min)
2B	17	2500 m/s	$\frac{1}{2}$ m	$\frac{1}{4}$ m	$\frac{1}{2}$ km	(5,1$\frac{1}{4}$ h)	(1,15 min)

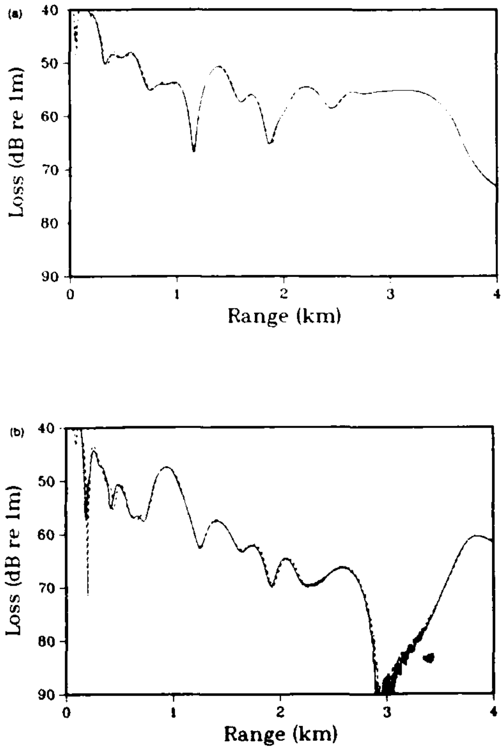

FIG. 2. The wedge with lossless sediment in cylindrical geometry. Transmission loss at depth (a) 30 m and (b) 150 m. The solid curve is the PE_2 result. The dashed curve is the PE_1 result.

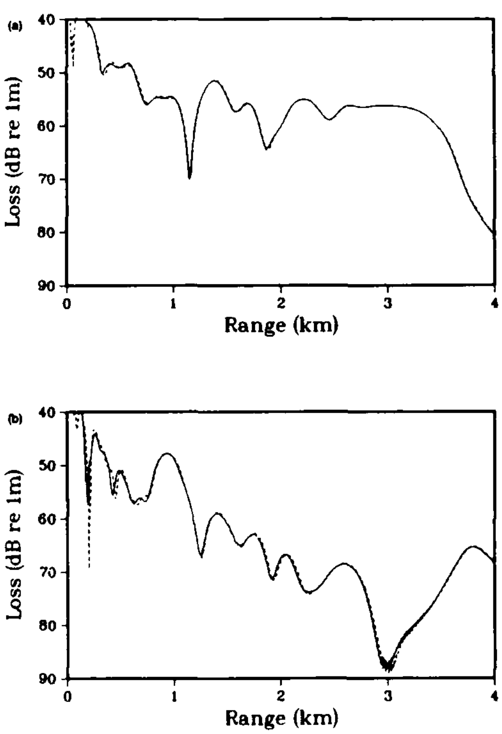

FIG. 3. The wedge with lossy sediment in cylindrical geometry. Transmission loss at depth (a) 30 m and (b) 150 m. The solid curve is the PE_2 result. The dashed curve is the PE_1 result.

that the run times required to produce data at many receiver depths (to produce contour plots) with COUPLE were more than ten times larger than the run times required to produce data at one receiver depth (to produce a transmission loss curve). This example illustrates that methods based on spectral decomposition can be inefficient if the solution is desired at many points in the domain. Normal-mode models are usually used for range-independent propagation problems when relatively few receivers are involved. When the solution is desired over the entire domain (this is the case for matched-field signal processing), however, it is possible that PE models are more efficient.

Problem 2 consists of a parallel waveguide in cylindrical geometry with a pressure release surface, a rigid bottom, and the sound speed

$$c(r,z) = (1500\ \text{m/s})/\sqrt{1 + \alpha E + \beta E^2 + \gamma E^3 + \delta E^4}, \tag{10}$$

$$\alpha = -(2\pi h_1/H)\cos(\pi z/H), \tag{11}$$

$$\beta = (\pi h_1/H)^2 - (4\pi h_2/H)\cos(2\pi z/H), \tag{12}$$

$$\gamma = (4\pi^2 h_1 h_2/H^2)\cos(\pi z/H), \tag{13}$$

$$\delta = (2\pi h_2/H)^2, \tag{14}$$

$$E = \exp(-\pi r/H), \tag{15}$$

where $h_1/H = 0.032$ and $h_2/H = 0.016$. Since the range dependence of the sound speed becomes more gradual with range, we update the sound-speed profile every range step for $r < 1$ km and every tenth range step for $r > 1$ km. Two cases were originally posed for this problem. However, we consider only the 25-Hz case with $z_0 = 250$ m and $H = 500$ m. Following Ref. 15, we divide this problem into part A, for which the first ten modes are excited, and part B, for which all 17 modes are excited.

Transmission loss at $z = 250$ m generated with PE_1 and PE_2 appears in Fig. 4 for part A. The PE_1 result agrees with the PE_1 result of Ref. 15, and the PE_2 result agrees with the PE_∞ result of Ref. 15. Transmission loss at $z = 250$ m appears in Fig. 5 for part B. The PE_1 result agrees with the PE_1 result of Ref. 15, and the PE_5 result agrees well with the PE_∞ result of Ref. 15. A large value was used for c_0 for part B due to the large phase velocities of the higher modes.

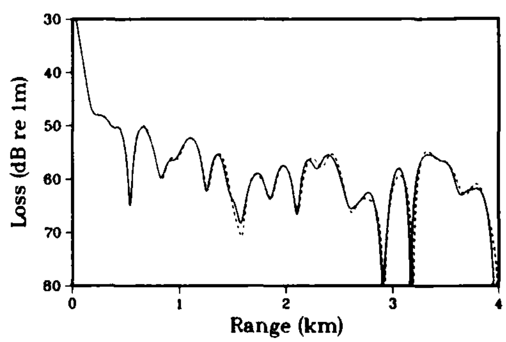

FIG. 4. The parallel waveguide in cylindrical geometry with range-dependent profile and ten modes excited. Transmission loss at depth 250 m. The solid curve is the PE_2 result. The dashed curve is the PE_1 result.

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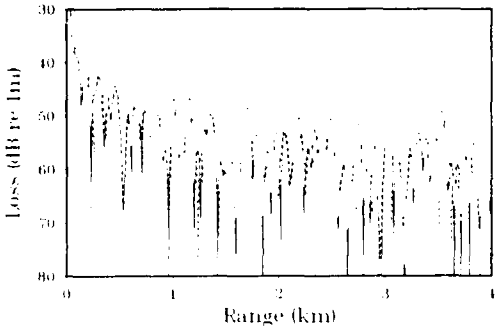

FIG. 5. The parallel waveguide in cylindrical geometry with range-dependent profile and 17 modes excited. Transmission loss at depth 250 m. The solid curve is the PE_5 result. The dashed curve is the PE_1 result.

III. CONCLUSION

The higher-order PE of Bamberger *et al.* produces results that agree well with outgoing coupled-mode results, even for propagation nearly orthogonal to the preferred direction. Greene's wide-angle PE starter appears to be accurate for a larger aperture than Claerbout's PE. Due to an improved algorithm for solving tridiagonal systems, the FEPE model is several times faster than the IFDPE model, especially for problems involving sloping bathymetry.

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